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Cockcroft Institute Postgraduate Lectures
Numerical Methods and Lattice Design

Lecture 4: Linear Optics and Matrices

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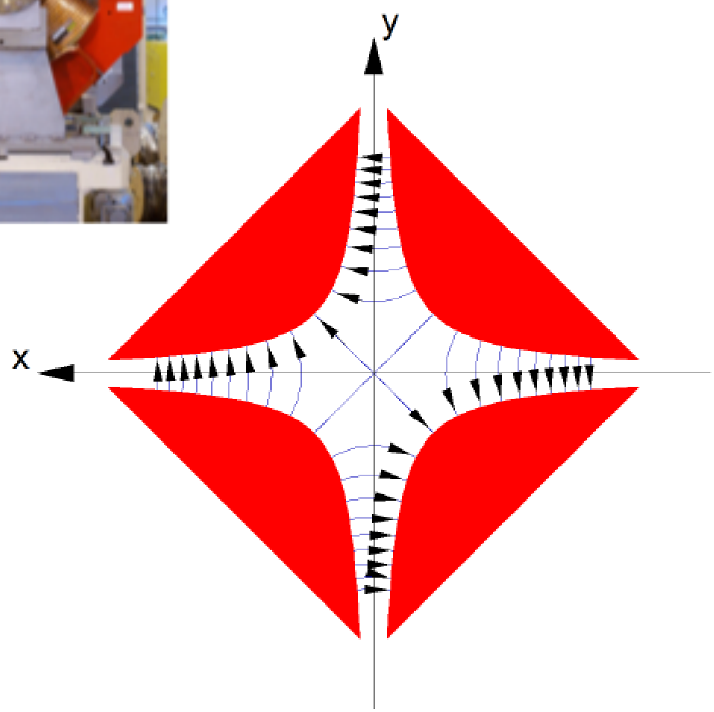
The Course Syllabus and Projects

- Recap on programming languages for physics; MATLAB and Python; summary of commands;
- Introduction to numerical computing; errors in computer calculations;
- Numerical integration methods; Euler's method; higher-order methods;
- Precision vs. accuracy; validation;
- Phase space; conserved quantities;
- Introduction to mappings and nonlinear systems;
- *Example: Methods for solving the linear and non-linear simple harmonic oscillator.*
- Introduction to Monte Carlo methods; Monte Carlo integration; classical problems;
- Pseudorandom and quasirandom sampling; methods of sampling; generation of distributions;
- Particle transport simulation; nuclear cross sections; particle histories; applications of Monte Carlo transport;
- *Example: Simulation of penetration of neutrons through shielding.*
- From mappings to linear optics; the concept of lattices;
- Transfer matrices and periodic solutions; propagation of linear optics parameters;
- Classic optical systems: the FODO, the double-bend achromat;
- Matching and optimisation; penalty/objective functions;
- Hill-climbing methods: Cauchy's method, Nelder-Mead, simulated annealing;
- Variables and constraints; under- and over-constrained problems;
- *Example: MAD8 matching of FODO Twiss values;*
- Multiple-configuration methods; genetic algorithms and evolutionary algorithms;
- A bestiary of codes; choosing the right code;
- Common pitfalls;
- *Example: Particle tracking in MAD8;*

SHM In Linear Optical Systems

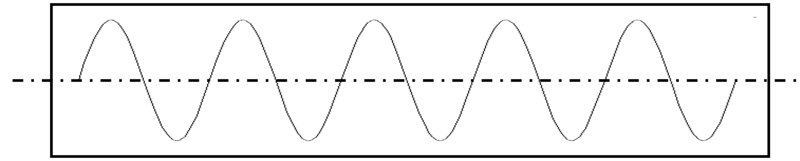
Nicholas Christofilos
Greek Elevator Engineer and
Amateur Accelerator Physicist



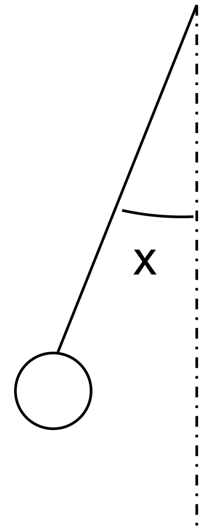
Normal quadrupole

$$B_x = b_2 \frac{y}{r_0}, \qquad B_y = b_2 \frac{x}{r_0}.$$

SHM through a long quadrupole

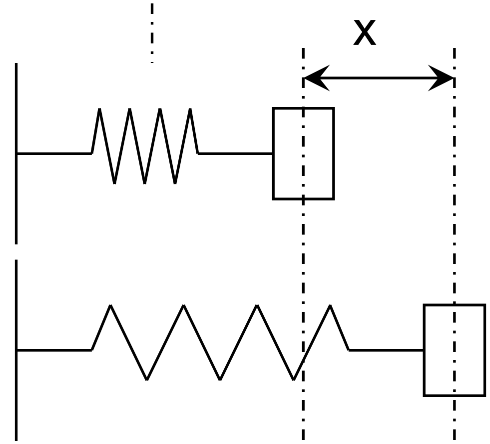


Physical analogies



Pendulum
(small angles only!)

Spring-mass



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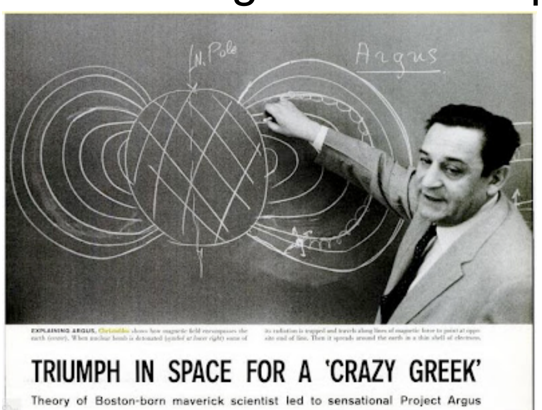
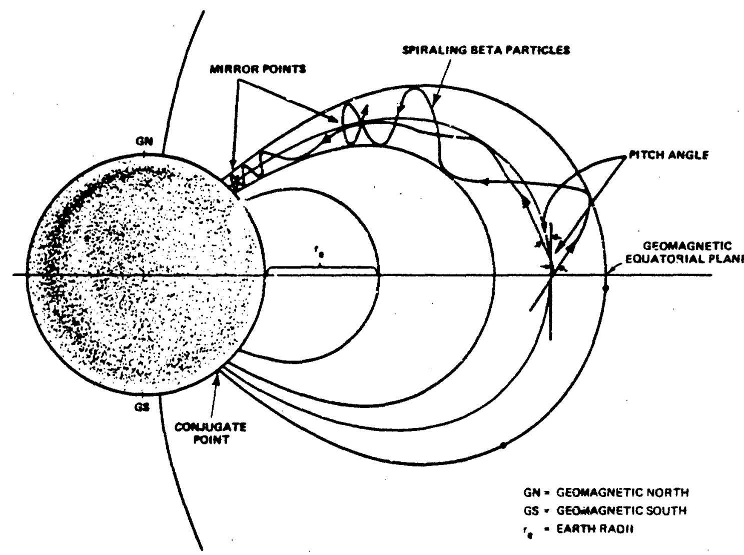
Nick Christofilos and Particles in Space

Starfish Prime (1.4 MT) a.k.a. the 'Rainbow Bomb' - knocked out Ariel 1, the first British satellite, and Telstar 1, the first TV satellite

..turning the Earth into a giant particle accelerator, to knock out incoming nuclear weapons





Artificial 'Beta injection' into inner radiation belt; particles remained trapped for 5 years!

Weak Focusing

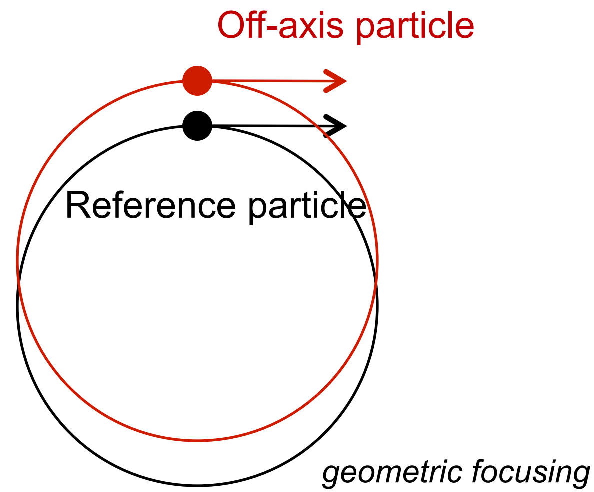
Consider a constant energy particle in a fixed dipole field (into the page)

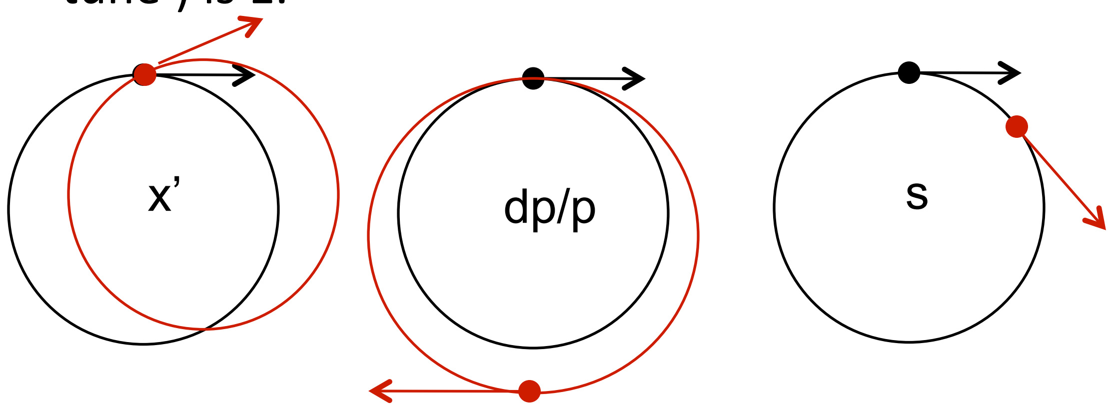
Reference particle executes cyclotron motion

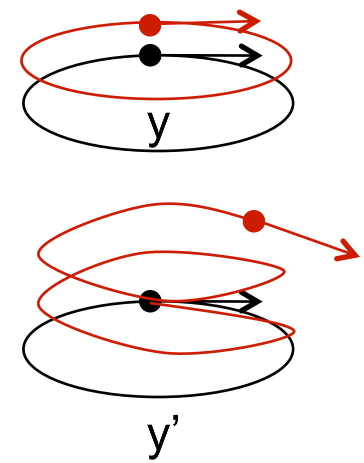
Other particles of the same energy execute cyclotron motion of same radius

Compared to reference trajectory, this looks like an oscillation

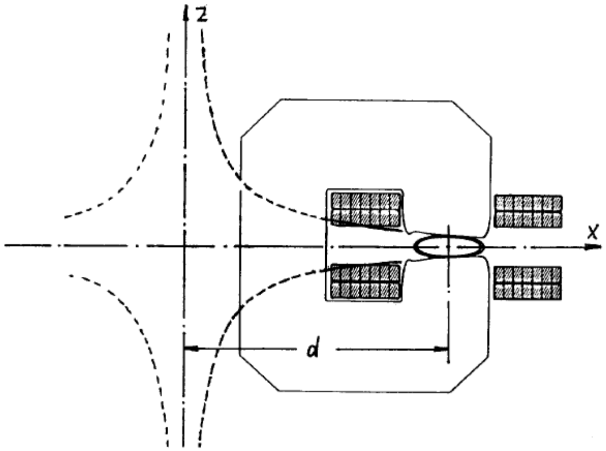
Number of oscillations per turn (the ‘tune’) is 1.

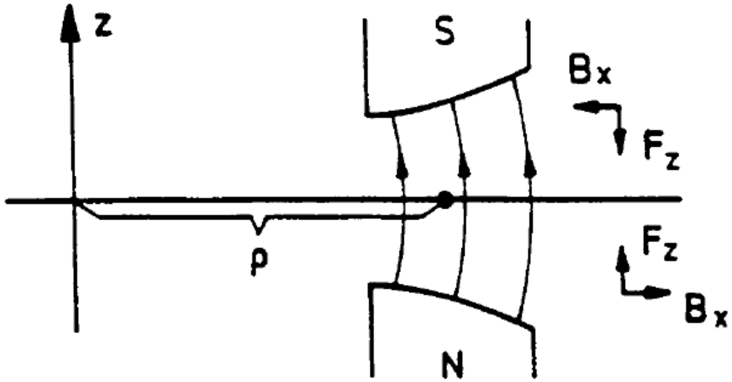




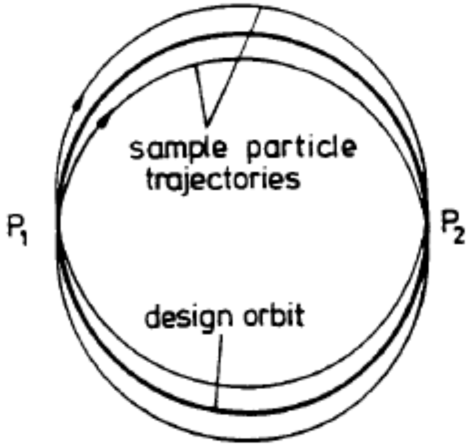


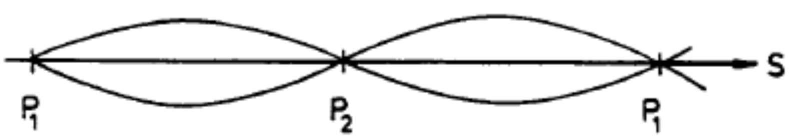
Stability in a Combined Function Magnet





$$n = -\frac{\rho}{B_0}\left(\frac{\partial B_z}{\partial r}\right)_{r=\rho}$$

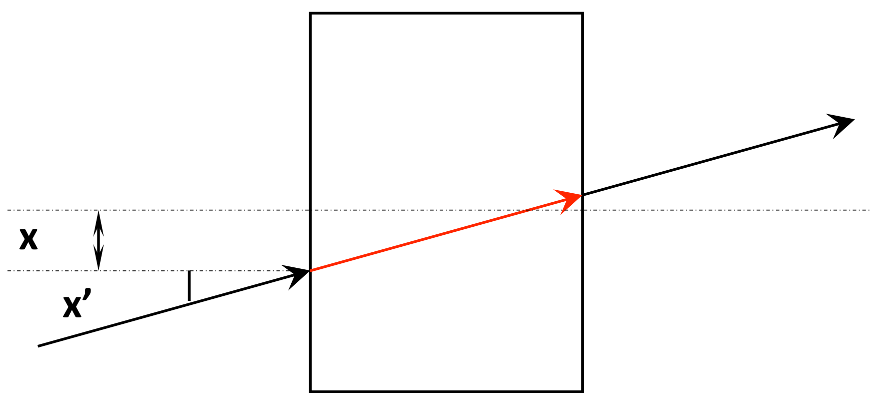


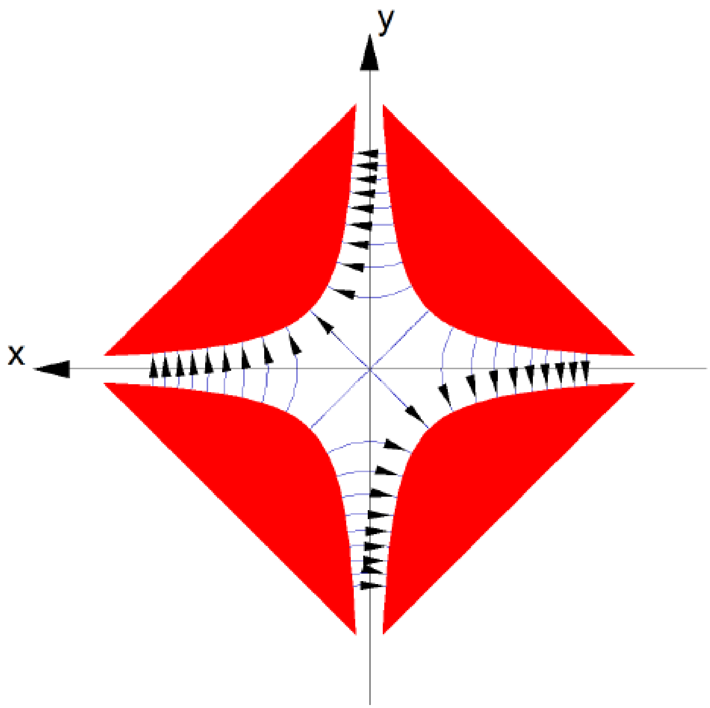


The 6 Co-Ordinates and the Linear Drift Space

$$\vec{x} = \begin{pmatrix} x \\ p_x \\ y \\ p_y \\ z \\ \delta \end{pmatrix} \qquad R = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x_1 = R.x_0$$





Normal quadrupole

$$B_x = b_2 \frac{y}{r_0}, \qquad B_y = b_2 \frac{x}{r_0}.$$

$$k_1 = \frac{q}{P_0} \frac{b_2}{r_0}$$

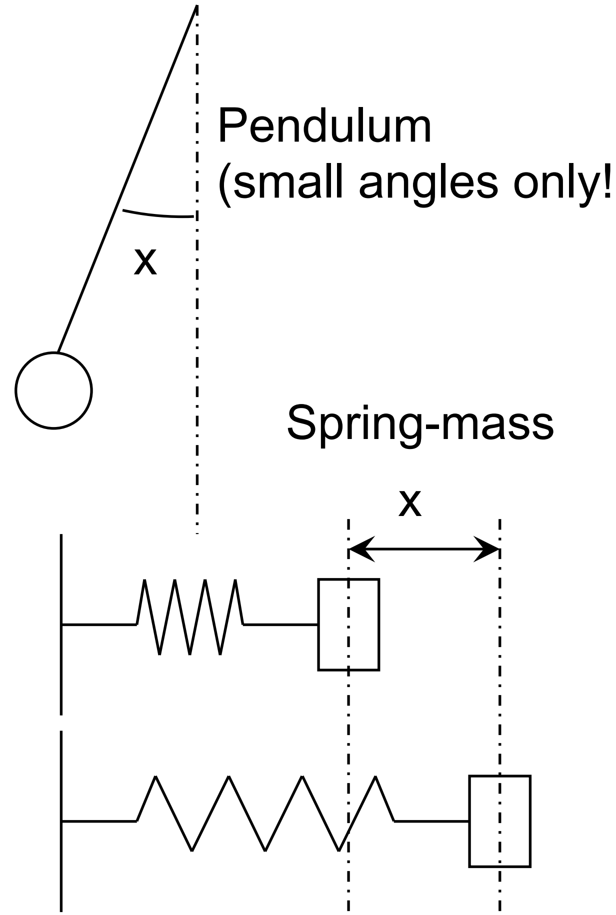
Normalised field gradient
(this is the number used in nearly all codes)
Doesn't depend on energy

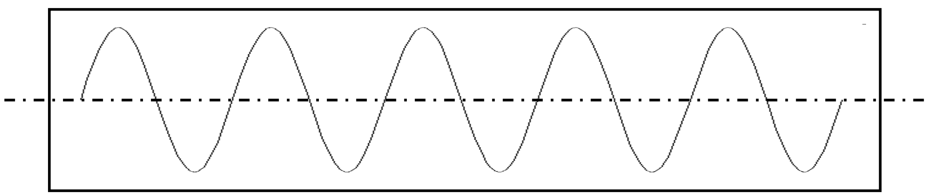
$$R = \begin{pmatrix} \cos\omega L & \frac{\sin\omega L}{\omega} & 0 & 0 & 0 & 0 \\ -\omega\sin\omega L & \cos\omega L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh\omega L & \frac{\sinh\omega L}{\omega} & 0 & 0 \\ 0 & 0 & \omega\sinh\omega L & \cosh\omega L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\omega = \sqrt{k_1}$$

$$R = \begin{pmatrix} \cos\omega L & \frac{\sin\omega L}{\omega} & 0 & 0 & 0 & 0 \\ -\omega\sin\omega L & \cos\omega L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh\omega L & \frac{\sinh\omega L}{\omega} & 0 & 0 \\ 0 & 0 & \omega\sinh\omega L & \cosh\omega L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Physical analogies

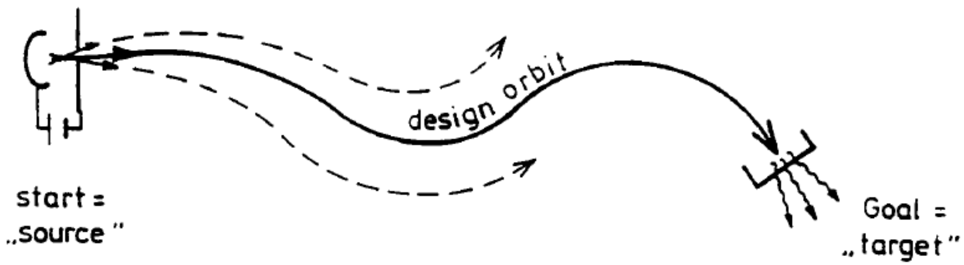


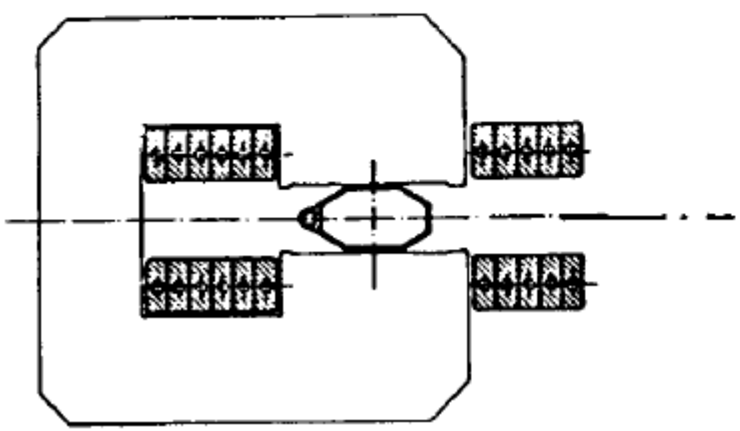


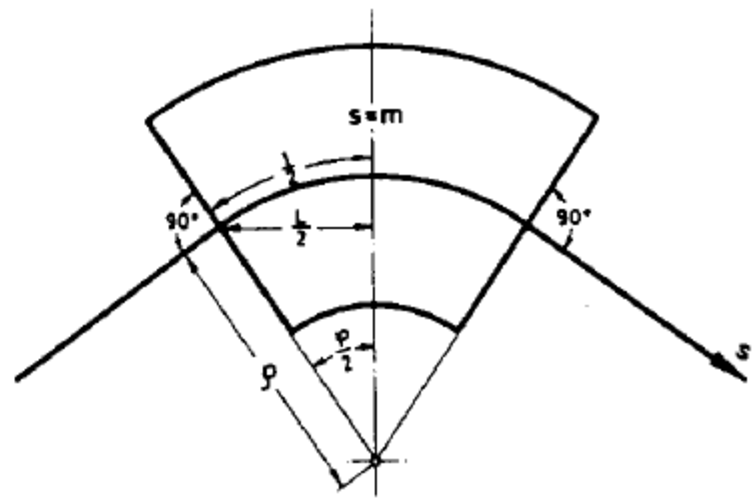
A particle executes simple harmonic motion in an infinitely long quadrupole

Transversely, an accelerator is analogous to a pendulum where gravity changes as a function of time

The Sector Bend – Assumption of a Curved Co-ordinate System







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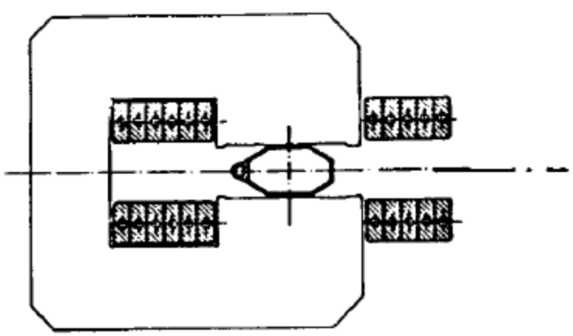
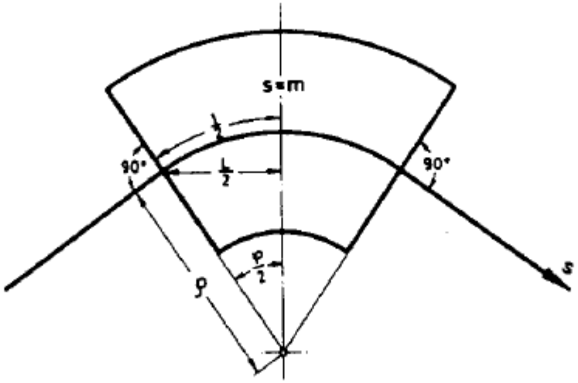
The Linear Wedge Dipole

$$R = \begin{pmatrix} \cos\omega L & \frac{\sin\omega L}{\omega} & 0 & 0 & 0 & \frac{1-\cos\omega L}{\omega\beta_0} \\ -\omega\sin\omega L & \cos\omega L & 0 & 0 & 0 & \frac{\sin\omega L}{\beta_0} \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\sin\omega L}{\beta_0} & -\frac{1-\cos\omega L}{\omega\beta_0} & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} - \frac{\omega L - \sin\omega L}{\omega\beta_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\omega = k_0 = \frac{q}{P_0} B_0$$





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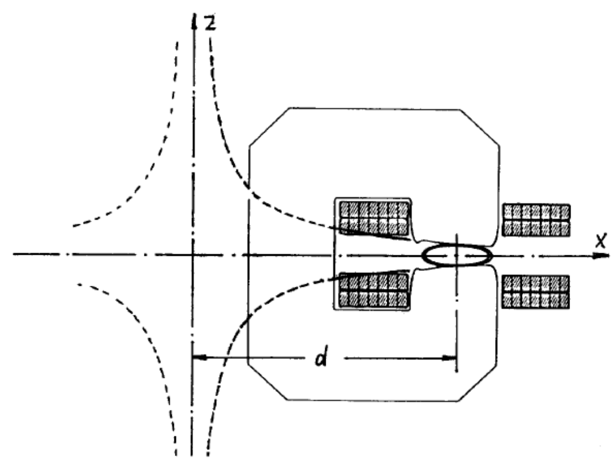
Linear Wedge Dipole with Focusing

$$R=\begin{pmatrix}
\cos\omega_x L & \frac{\sin\omega_x L}{\omega_x} & 0 & 0 & 0 & \frac{k_0}{\beta_0}\frac{(1-\cos\omega_x L)}{\omega_x^2} \\
-\omega_x\sin\omega_x L & \cos\omega_x L & 0 & 0 & 0 & \frac{k_0}{\beta_0}\frac{\sin\omega_x L}{\omega_x} \\
0 & 0 & \cosh\omega_y L & \frac{\sinh\omega_y L}{\omega_y} & 0 & 0 \\
0 & 0 & \omega_y\sinh\omega_y L & \cosh\omega_y L & 0 & 0 \\
-\frac{k_0}{\beta_0}\frac{\sin\omega_x L}{\omega_x} & -\frac{k_0}{\beta_0}\frac{(1-\cos\omega_x L)}{\omega_x^2} & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2}-\frac{k_0^2}{\beta_0^2}\frac{(\omega_x L-\sin\omega_x L)}{\omega_x^3} \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$\omega_x=\sqrt{k_0^2+k_1}, \qquad \omega_y=\sqrt{k_1}$$

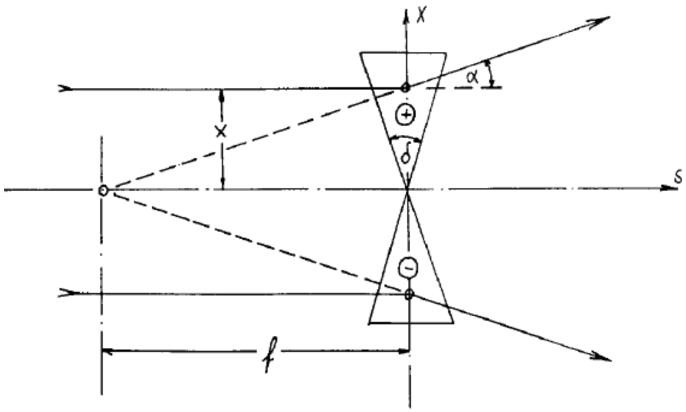
$$B_x=b_2\frac{y}{r_0}, \qquad B_y=b_1+b_2\frac{x}{r_0}$$

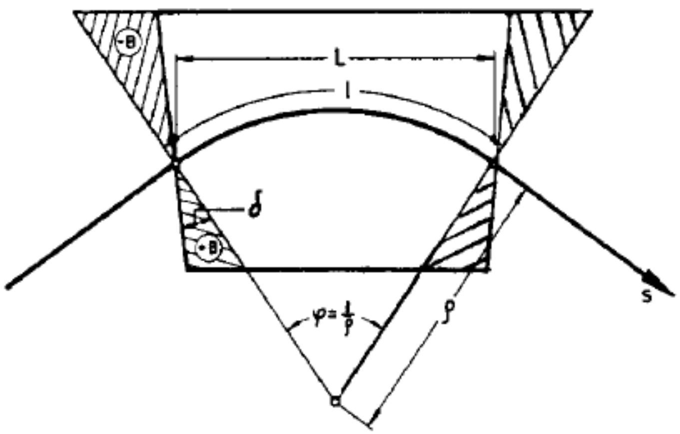
$$k_0=\frac{q}{P_0}b_1, \qquad k_1=\frac{q}{P_0}\frac{b_2}{r_0}$$



$$x_1=R.x_0$$

Edge Focusing

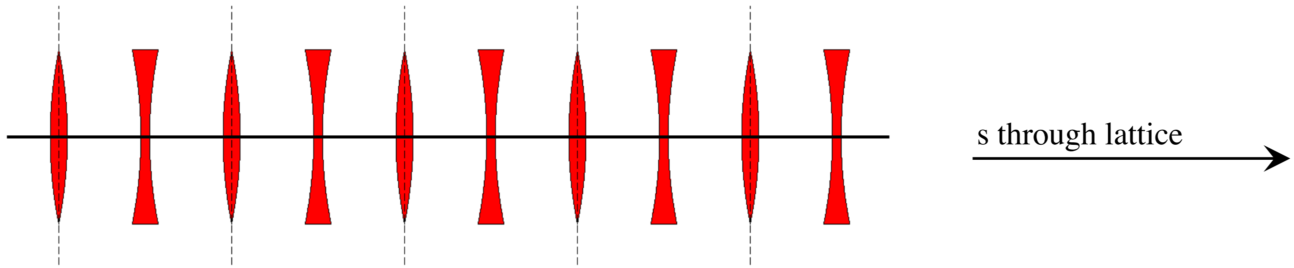




$$R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -K_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & K_1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_1 = -\frac{q}{P_0} B_0 \tan\psi$$

The Fodo Lattice

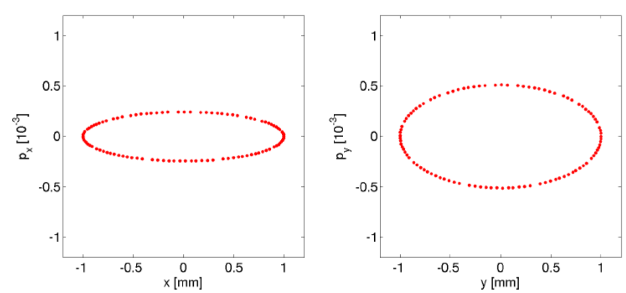


$L \to 0 \qquad k_1 L \to \frac{1}{f}$ (thin-lens approximation)

$$R_Q(f) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1/f & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/f & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad R_D(L) = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R = R_Q(2f_0) \cdot R_D(L) \cdot R_Q(-f_0) \cdot R_D(L) \cdot R_Q(2f_0)$$

$$R = \begin{pmatrix} 1 - \frac{L^2}{2f_0^2} & \frac{L}{f_0}(L+2f_0) & 0 & 0 & 0 & 0 \\ \frac{L}{4f_0^3}(L-2f_0) & 1 - \frac{L^2}{2f_0^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - \frac{L^2}{2f_0^2} & -\frac{L}{f_0}(L-2f_0) & 0 & 0 \\ 0 & 0 & -\frac{L}{4f_0^3}(L+2f_0) & 1 - \frac{L^2}{2f_0^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{2L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



(hint: this is a Poincare section)

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From rays to Twiss values

$$R = \begin{pmatrix} 1-\frac{L^2}{2f_0^2} & \frac{L}{f_0}(L+2f_0) & 0 & 0 & 0 & 0 \\ \frac{L}{4f_0^3}(L-2f_0) & 1-\frac{L^2}{2f_0^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\frac{L^2}{2f_0^2} & -\frac{L}{f_0}(L-2f_0) & 0 & 0 \\ 0 & 0 & -\frac{L}{4f_0^3}(L+2f_0) & 1-\frac{L^2}{2f_0^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{2L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

It can be shown that: $R^{\mathsf{T}} \cdot S \cdot R = S$

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

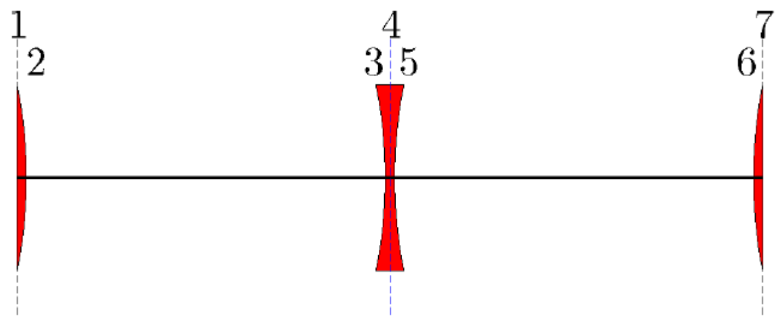
w.l.o.g. we can write: $R_2 = I_2 \cos\mu_x + S_2 \cdot A_2 \sin\mu_x$

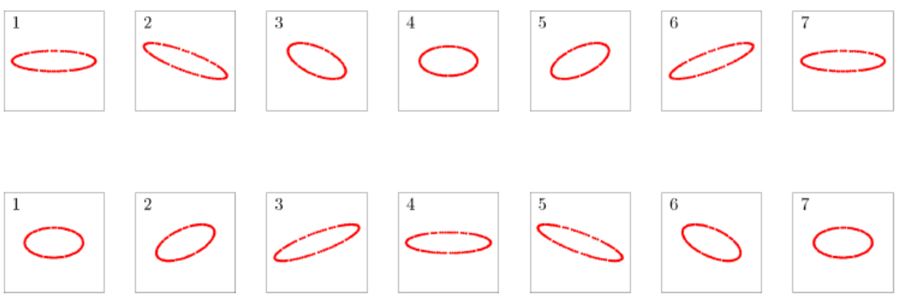
with: $A_2 = \begin{pmatrix} \gamma_x & \alpha_x \\ \alpha_x & \beta_x \end{pmatrix}$

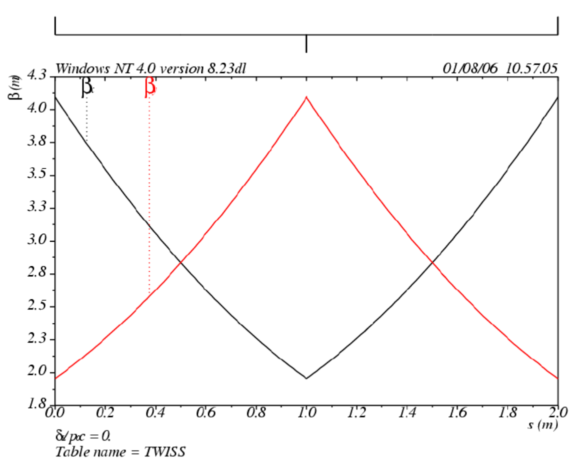
which gives: $\beta_x\gamma_x - \alpha_x^2 = 1$ and $R_2^{\mathsf{T}} \cdot A_2 \cdot R_2 = A_2$

- So what?
  - Well, this means that the values of alpha, beta, gamma are constants of the accelerator lattice, and not of any particular particle
  - Beta in particular describes the envelope of the particles
  - Alpha, beta, gamma are the Twiss functions (functions of ‘s’).

Propagation of the Twiss values







$$A_2^{-1}(s_1) = R_2(s_0, s_1) \cdot A_2^{-1}(s_0) \cdot R_2^{\mathrm{T}}(s_0, s_1)$$

where: $$A_2^{-1}(s) = \begin{pmatrix} \beta_x(s) & -\alpha_x(s) \\ -\alpha_x(s) & \gamma_x(s) \end{pmatrix}$$

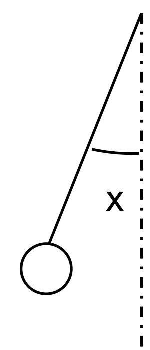
$$\begin{aligned} x &= \sqrt{2\beta_x J_x} \cos \phi_x \\ p_x &= -\sqrt{\frac{2J_x}{\beta_x}} (\sin \phi_x + \alpha_x \cos \phi_x) \end{aligned}$$

$$J_x = \frac{1}{2}\left(\gamma_x x^2 + 2\alpha_x x p_x + \beta_x p_x^2\right)$$

is a conserved quantity

$$\Delta\phi_x = \int_{s_1}^{s_2} \frac{1}{\beta_x} ds$$

$$\nu_x = \frac{\Delta\phi_x}{2\pi} = \frac{1}{2\pi} \int_0^{C_0} \frac{1}{\beta_x} ds$$



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The Periodic Solution

$$R_2 = \begin{pmatrix} \cos\mu_x + \alpha_x \sin\mu_x & \beta_x \sin\mu_x \\ -\gamma_x \sin\mu_x & \cos\mu_x - \alpha_x \sin\mu_x \end{pmatrix}$$

This is always the form of the 1-turn matrix

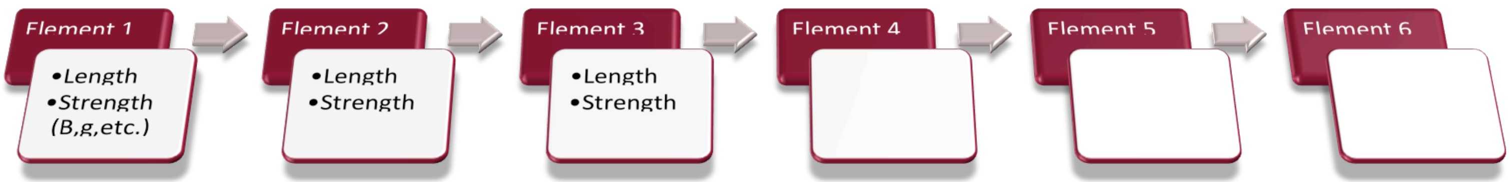
$$R_2 = \begin{pmatrix} 1 - \frac{L^2}{2f_0^2} & \frac{L}{f_0}(L + 2f_0) \\ \frac{L}{4f_0^3}(L - 2f_0) & 1 - \frac{L^2}{2f_0^2} \end{pmatrix}$$

This is the particular form for our FODO example

- For a real-valued phase advance (and therefore real-valued Twiss functions), the Trace of the transfer matrix must be less than 2.
  - Don't forget the other plane of motion – you can be stable in one plane and not in the other…
  - In other words, you have to get the focusing strengths right.

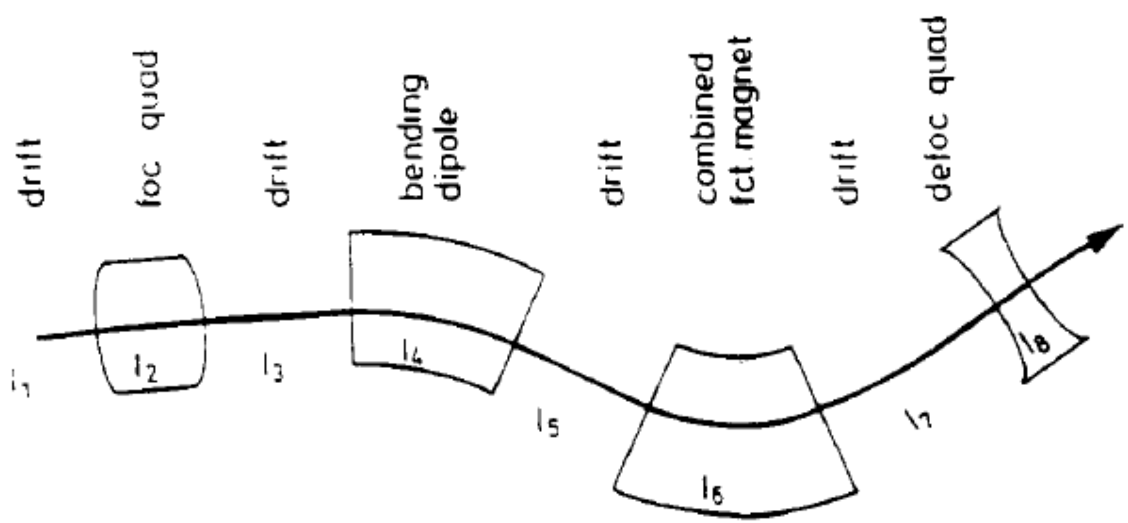
So how do we do it in a code?

- Accelerator codes simply assume a piecewise-continuous representation of the accelerator structure.



$$R_T = R_n.R_{n-1}...R_3.R_2.R_1$$

- but because of edge focusing the number of matrices is **not** the same as the number of elements.



Calculation of the Twiss values – periodic solution procedure

- Linear optics codes work using the following procedure:

1. Parse lattice structure into R matrices
2. Calculate one-turn matrix: $R_T = R_n.R_{n-1}...R_3.R_2.R_1$
3. Determine stability in each plane using $Tr(R_2) < 2$

$$R_2 = \begin{pmatrix} \cos\mu_x + \alpha_x \sin\mu_x & \beta_x \sin\mu_x \\ -\gamma_x \sin\mu_x & \cos\mu_x - \alpha_x \sin\mu_x \end{pmatrix}$$

4. Calculate the periodic phase advance: $\mu = arccos[(R_{11} + R_{22})/2]$
5. Calculate the initial Twiss values: $\beta_0 = \frac{R_{12}}{sin(\mu)}$ $\alpha_0 = \frac{R_{11} - R_{12}}{2sin(\mu)}$ $\gamma_0 = \frac{1+\alpha_0^2}{\beta_0}$

   (Exercise: what happens when β_0 comes out negative?)

6. Propagate the Twiss values.

Propagating Twiss and phase values

- This is easy…

$$T = A_2^{-1} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

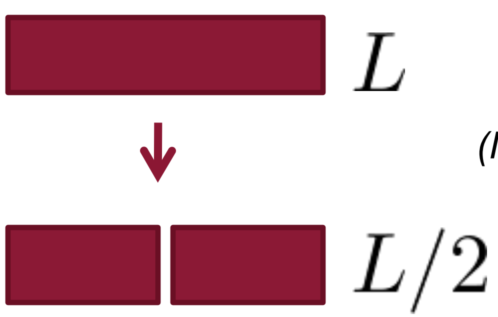
$$T_2 = R_{21}.T_1.R_{21}^T$$

- and phases are propagated as

$$\mu_2 = arctan\left(\frac{R_{12}}{\beta_1 R_{11} - \alpha_1 R_{12}}\right)$$

- the important thing to note here is that elements can be split… (but the edge focusing is not split over dipoles)

T_1, μ_1



(NB: don't have to split for the one-turn matrix)

(hint: you can put other calculations in when you do this)

Quiz Question

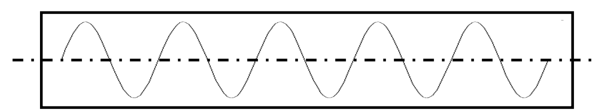
- What is the horizontal beta function in a classical cyclotron?

Continuous verus discrete calculations

- Most accelerator physics courses talk about **continuous** functions.
- Of course, virtually all codes calculate **discrete** values using the linear matrix formalism.
- Depending on the code and/or the particular lattice, you can have problems. Most of these problems arise because of the oscillatory form of the trajectory solutions.

- **Example**: Betatron tune through a long quadrupole
- Continuously, we have $$\nu(s) = \int_0^s \frac{ds}{\beta(s)}$$
- but in a discrete quadrupole the matrix calculation loses the integer part of the phase advance.

$$R = \begin{pmatrix} \cos\omega L & \frac{\sin\omega L}{\omega} & 0 & 0 & 0 & 0 \\ -\omega\sin\omega L & \cos\omega L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh\omega L & \frac{\sinh\omega L}{\omega} & 0 & 0 \\ 0 & 0 & \omega\sinh\omega L & \cosh\omega L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



- Few codes check for this!

Dispersion

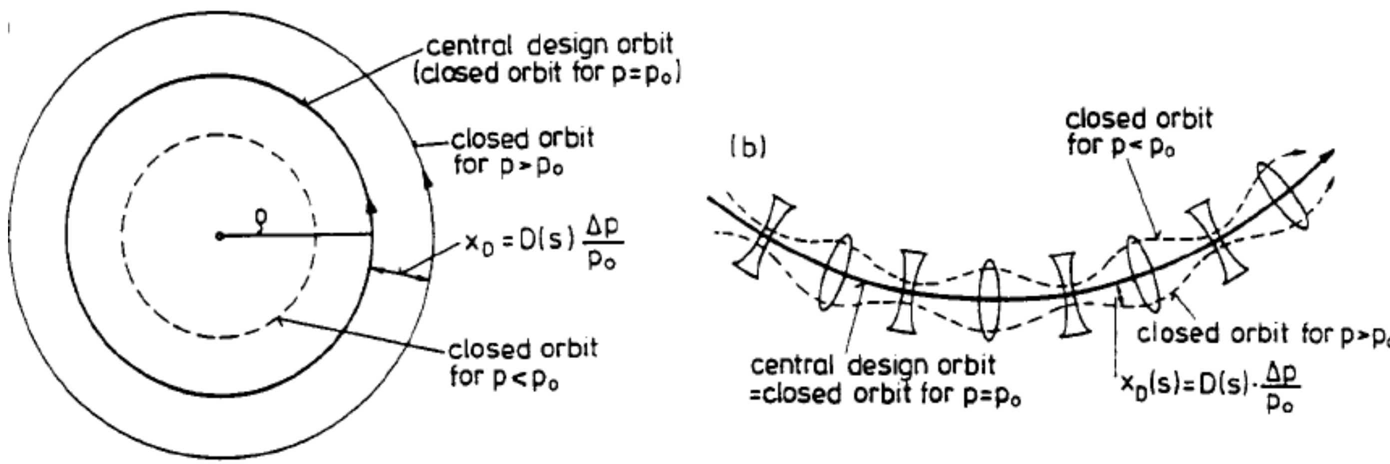
- Dispersion is actually much easier than it might first appear.

- The dispersion function is that ray which has unit momentum deviation, i.e. $\Delta p/p = 1$

- In other words, at any location through a lattice it can be described as a vector of the form:

$$\vec{D} = \begin{pmatrix} \eta_x \\ \eta_x' \\ \eta_y \\ \eta_y' \\ 0 \\ 1 \end{pmatrix}$$

- Assuming a linear system, all other energies just scale from this value.



Propagating dispersion

- Propagating dispersion is even easier…

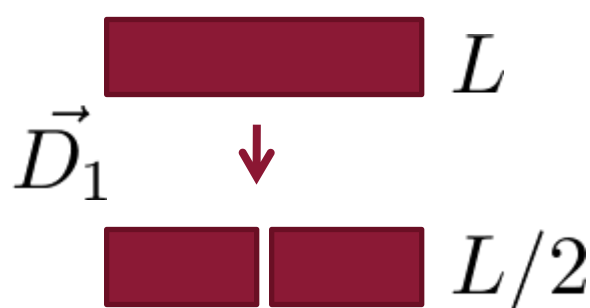
$$\vec{D}_2 = R_{21}.\vec{D}_1$$

(remember, the dispersion is just another ray/particle)

- For a periodic system with one-turn 6x6 matrix R, it can be shown (!) that the periodic solution is:

$$\vec{D}_0 = \begin{pmatrix} \frac{(1-R_{22})R_{16}+R_{12}R_{26}}{(1-R_{11})(1-R_{22})-R_{21}R_{12}} \\ \frac{(1-R_{11})R_{26}+R_{21}R_{16}}{(1-R_{11})(1-R_{22})-R_{21}R_{12}} \\ \frac{(1-R_{44})R_{36}+R_{34}R_{46}}{(1-R_{33})(1-R_{44})-R_{43}R_{34}} \\ \frac{(1-R_{33})R_{46}+R_{43}R_{36}}{(1-R_{33})(1-R_{44})-R_{43}R_{34}} \\ 0 \\ 1 \end{pmatrix}$$

- And again, we can split elements when propagating…



Optical modules

- There are many possible configurations of dipoles and quadrupoles that can give stable motion
  - In particular, we can talk about *dispersion-free* lattices, which are important in many applications
    - Colliders, SR sources
  - Chasman-green, double-bend achromat, triple-bend achromat…

- Nearly always, someone has worked out the rules for a typical optical module that does a particular job.
  - It then needs adapting to your particular problem, e.g. taking account of space, beam energy, technological limitations/choices etc.

- In the following slides we will see a few examples of such modules.

A non-dispersive bending system – the achromat

Example of nondispersive bending system

Φ = sector magnet bend. angle

$\varphi = \ell\sqrt{k}$ = quadrupole magnet phase angle

λ = drift space length

The system is nondispersive if the cosinelike trajectory (with respect to the central symmetry point) goes through the mid-point of the bending magnets, i.e. if

$$\frac{1}{\sqrt{k}} \operatorname{ctn}\frac{\varphi}{2} = \rho \tan\frac{\Phi}{2} + \lambda \, .$$

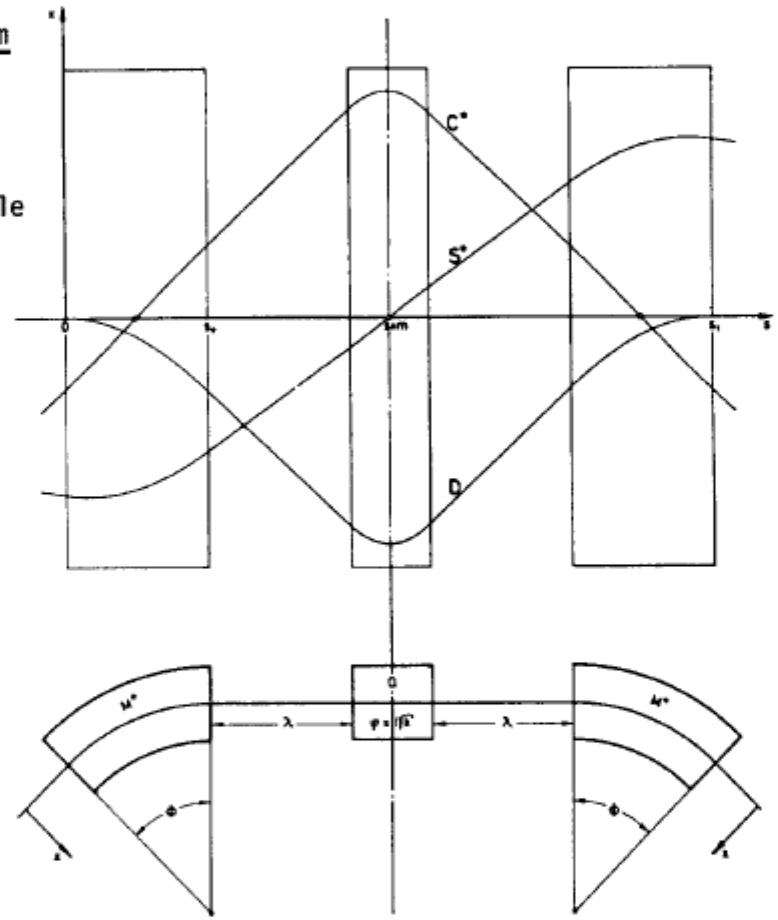


Fig. 14: Nondispersive deflecting system.

(from K. Steffen's excellent CAS lectures)

A non-dispersive translating system – the dog-leg

Example of nondispersive translating system

Φ = sector magnet bend. angle

$\varphi = \ell\sqrt{k}$ = quadrupole magnet phase angle

d, λ = drift space lengths.

The system is nondispersive if the sinelike trajectory (with respect to the central symmetry point) goes through the mid-point of the bending magnets, i.e. if

$$\rho \tan\frac{\Phi}{2} + \lambda = \frac{1}{\sqrt{k}} \, \frac{d\sqrt{k}\cos\varphi + 2\sin\varphi}{d\sqrt{k}\sin\varphi - 2\cos\varphi} .$$

Focusing also in the other plane may be obtained by adding a third quadrupole of opposite polarity at the symmetry point.

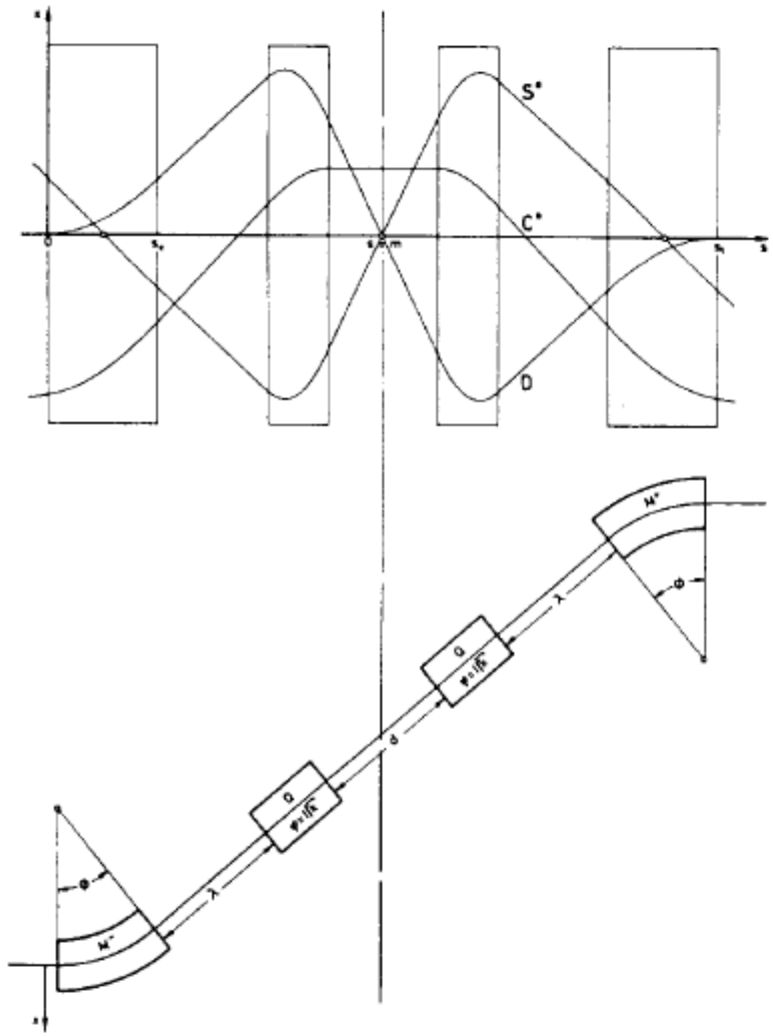


Fig. 15: Nondispersive translating system.

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A non-dispersive system using only dipoles

<u>Example of nondispersive sector magnet system</u>

Φ = sector magnet bending angle

$\frac{1}{\rho}$ = sector magnet bending strength

λ = drift space length.

The system is nondispersive for

$$\frac{\lambda}{\rho} = \frac{2\cos\Phi - 1}{\sin\Phi} = \mathrm{ctn}\,\Phi - \tan\frac{\Phi}{2}\,.$$

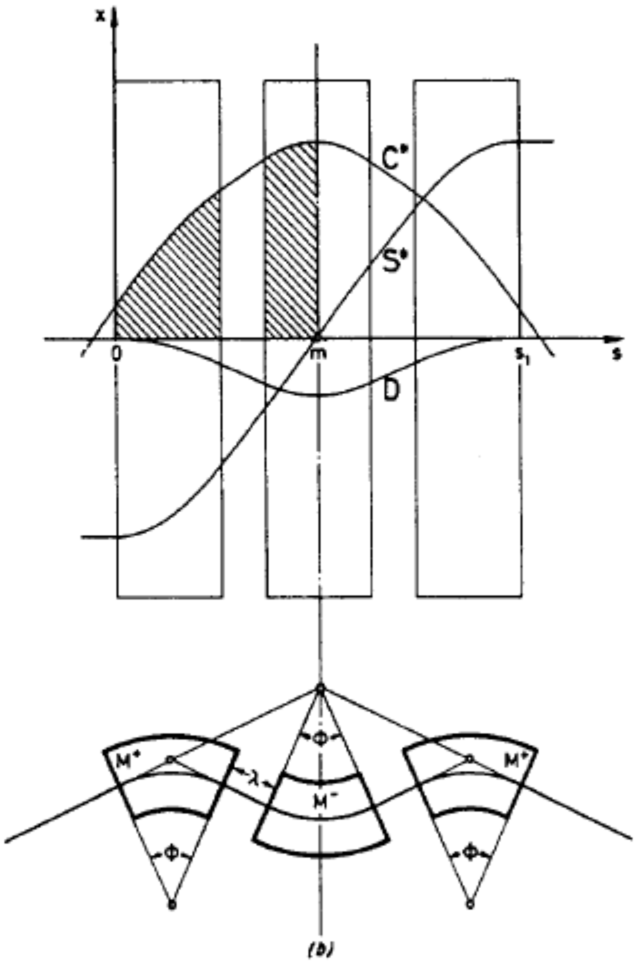


Fig. 17: Nondispersive sector magnet system.

A non-dispersive 4-magnet system – the chicane

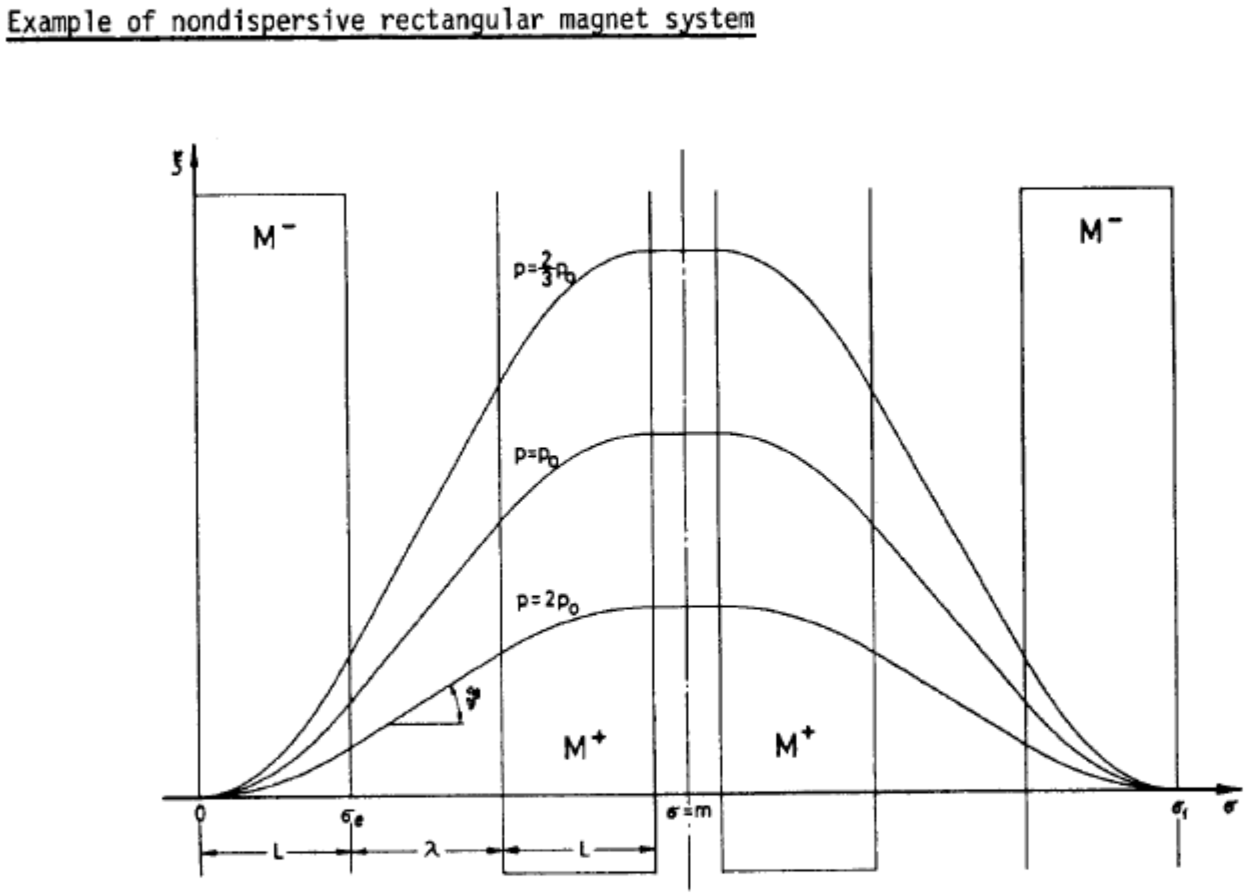


Fig. 18: Nondispersive momentum selecting system for large momentum spread.

Q: Why would you want one of these?

Exercise: derive the analytic value in such a system for R_{56}

An isochronous, achromatic bending system

Example of symmetric isochronous deflecting system

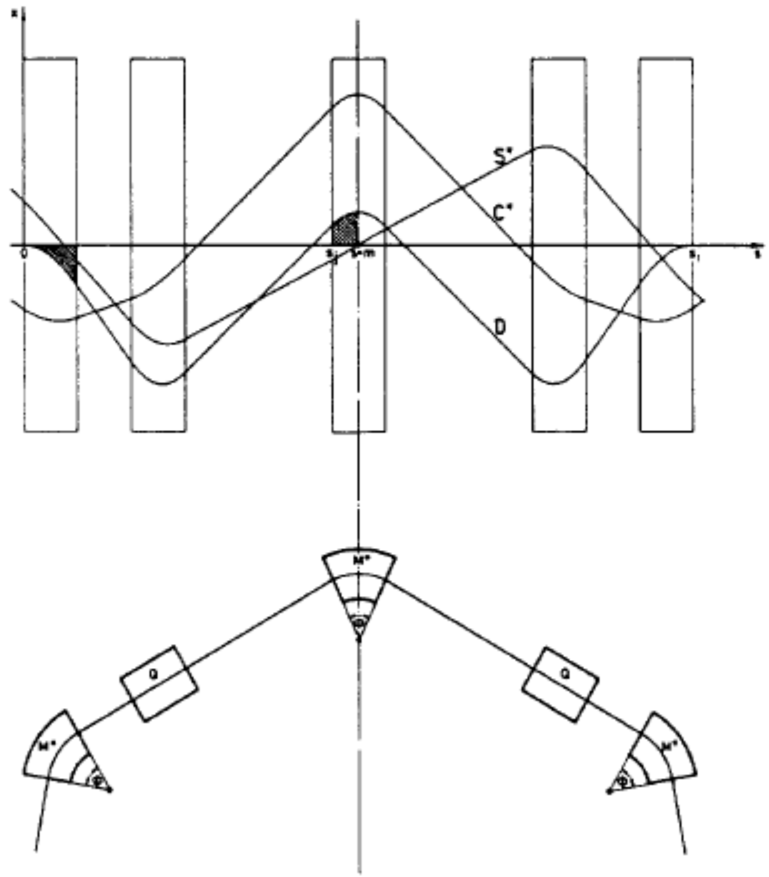


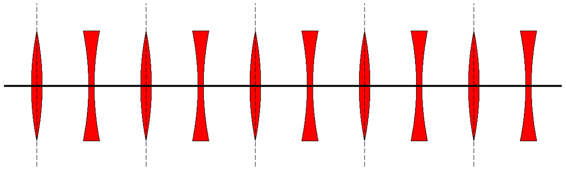
Fig. 19: Symmetric isochronous deflecting system.

Periodic and aperiodic systems

- There is no basic difference between a **periodic** and an **aperiodic** system.
- The propagation of individual rays is identical: $\vec{x_2} = R.\vec{x_1}$

- In an aperiodic system, initial Twiss and dispersion values must be supplied. *The values obtained when propagated will then differ depending upon the initial values.*

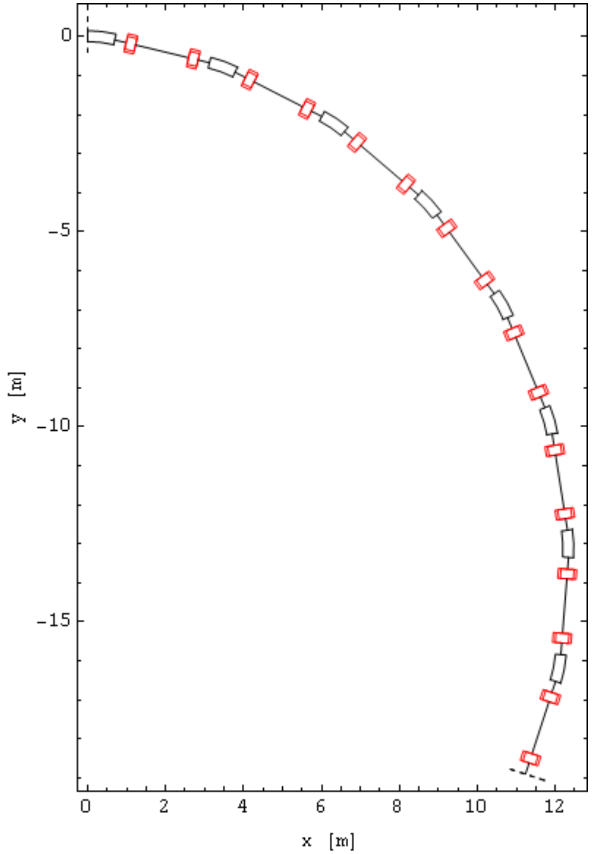
- Example: a FODO channel is set up which has a particular periodic solution for the Twiss values. We propagate an initial (different) Twiss matrix through the system – the propagated values are **different**. This is called **mismatch**.

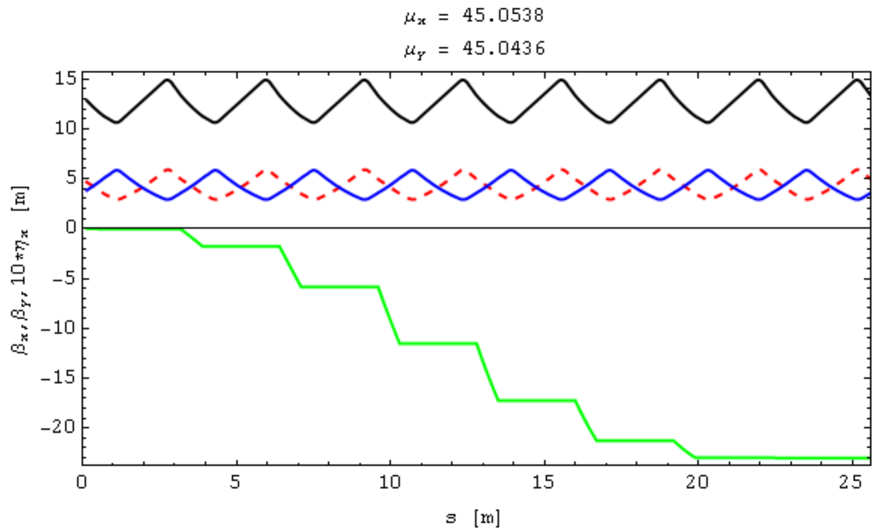


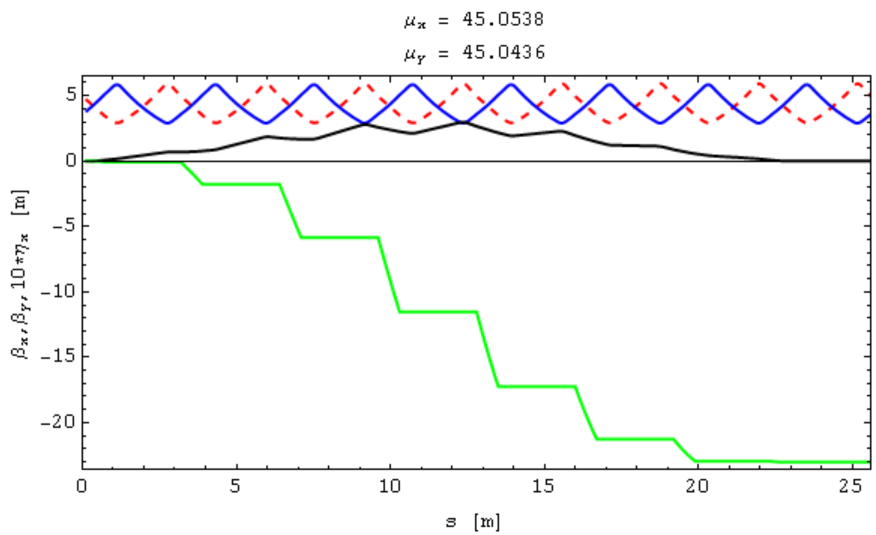
- Exercise: Try it out for yourself!

- The same thing is true of dispersion.

Deliberate dispersion mismatch – the dispersion suppressor







A FODO lattice with 45-degree phase advance

Real lattices, and lattice design

- Remember that when we design lattices, that eventually it will get built (hopefully!)

- Reality uses up more space than ideal elements….

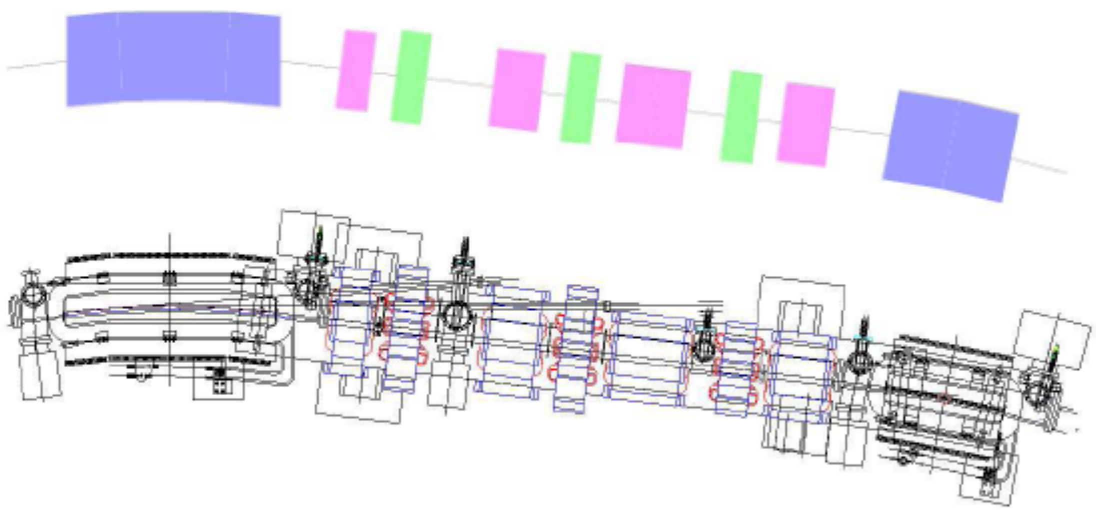


Fig. 1: Lattice sections as seen by the lattice designer (top) and the design engineer (bottom). Note how the space between ideal magnets is consumed by coils, beam-position monitors, absorbers, pumps, etc.

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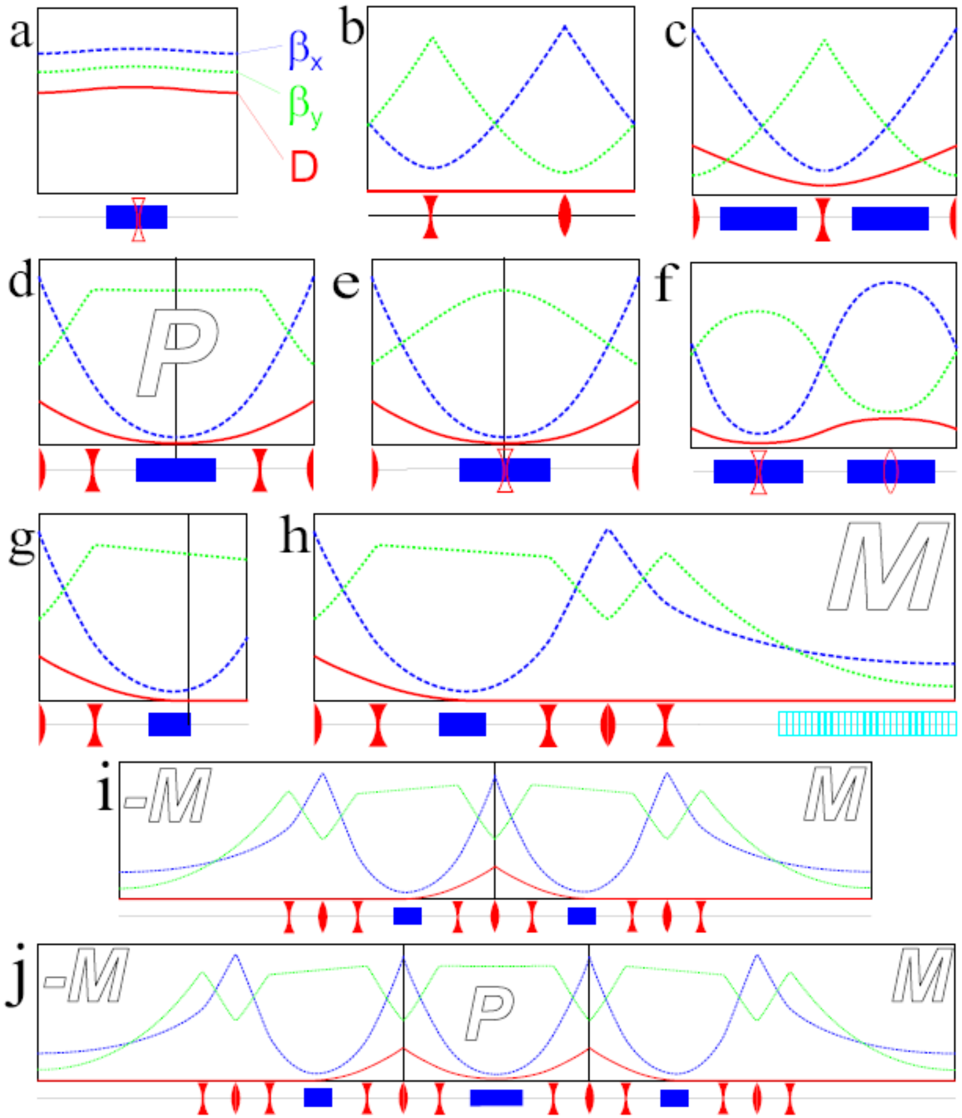
From simple to complex lattices



a: Weak focusing

b: FODO channel

c: FODO cell

d: Low-emittance cell

e: CF low-emittance cell

f: Low-emittance FODO

g: Dispersion match

h: Periodic dispersion match

i: Double-bend achromat

j: Triple-bend achromat

(see A.Streun's excellent course on low-emittance lattice design)

In the next lecture...

- In the next lecture, we will talk about taking an optical module, and adapting (i.e. improving) it for a particular purpose. This is often one of the principal jobs people use codes for.

- The general method is called **matching**.